Module 2:

1. Locate the stationary points, if any, and the global optimum of the following functions. Determine if each function is convex or concave.
   a. $f(X) = \text{[illegible]}x^4 + \text{[illegible]}x^3$
   b. $f(X) = \text{[illegible]}x^2 - \text{[illegible]}x + \text{[illegible]}$
   c. $f(X) = \text{[illegible]}^3 - \text{[illegible]} - \text{[illegible]}^2$
2. Locate the desired optimum of the following functions of multiple variables:
   a. $Minimize\ f(X) = \text{[illegible]}x_1^2 + \text{[illegible]}x_2^2 + \text{[illegible]}_1 x_2 - \text{[illegible]}x_1$
   b. $Minimize\ f(X) = \text{[illegible]}_1^2 + \text{[illegible]}_2^2 + \text{[illegible]}_1 x_2 + \text{[illegible]}_2^3$
   c. $Maximize\ f(X) = \text{[illegible]}x_1^3 - \text{[illegible]}_1^2 + \text{[illegible]}_1 x_2 - \text{[illegible]}x_2$
3. $Minimize\ f(X) = \text{[illegible]}x_1^2 + \text{[illegible]}x_2 - x_1 x_2 \qquad subject\ to\ x_1 + x_2 = \text{[illegible]}$
4. $Minimize\ f(X) = x_1^2 + x_2^2 \qquad subject\ to\ x_1 - x_2 = \text{[illegible]}$
5. $Optimize\ f(X) = -x_1^2 - x_2^2 + \text{[illegible]}x_1 + \text{[illegible]}x_2\ subject\ to\ x_1 + x_2 \le \text{[illegible]}\ and - \text{[illegible]}x_1 - \text{[illegible]}x_2 + 2 \ge \text{[illegible]}$
6. $Optimize\ f(X) = -\ \text{[illegible]}x^2 + \text{[illegible]}xy - \text{[illegible]}y^2 + 8x\ subject\ to\ x + y \le \text{[illegible]}$
7. $Minimize\ f(X) = \text{[illegible]}x_1 - \text{[illegible]})^2 + \text{[illegible]}x_2 - \text{[illegible]})^2\ subject\ to\ x_1 + \text{[illegible]}x_2 \le \text{[illegible]}\ and\ 8x_1 + \text{[illegible]}x_2 \ge \text{[illegible]}0$
8. From the solution obtained for problems 5, 6 and 7, check whether the Kuhn-Tucker conditions are satisfied.
9. Assume that the objective was to minimize the sum of squared deviations of the actual allocations x_j from some desired or known target allocations T_j. Given a supply of water Q less than the sum of all target allocations T_j, structure a planning model and its corresponding Lagrangian. Will a global minimum be obtained from solving the partial differential equations derived from the Lagrangian? Why?
10. Assume water can be allocated to three users. The allocation, x_j, to each use j provides the following returns: $R(x_1) = (12x_1 - x_1^2)$, $R(x_2) = (8x_2 - x_2^2)$ and $R(x_3) = (18x_3 - 3x_3^2)$. Assume that the objective is to maximize the total return, $F(\boldsymbol{X})$, from all three allocations and that the sum of all allocations cannot exceed 10. a) How much would each use like to have? b) Show that at the maximum total return solution the marginal values, $\partial(R(x_j))/\partial x_j$, are each equal to the shadow price or Lagrange multiplier (dual variable) λ associated with the constraint on the amount of water available. c) Finally, without resolving a Lagrange

multiplier problem, what would the solution be if 15 units of water were available to allocate to the three users and what would be the value of the Lagrange multiplier?