Module 5:

1. A reservoir is planned for irrigation and low flow augmentation for water quality control. A storage volume of 6×10^6 m^3 will be available for those two conflicting uses each year. The maximum irrigation demand (capacity) is 4×10^6 m^3. Let X_1 be the allocation of water to irrigation and X_2 the allocation for downstream flow augmentation. Assume that there are two objectives, expressed as

$$Z_1 = 4\,X_1 - X_2$$

$$Z_2 = -2\,X_1 + 6X_2$$

   (a) Write the multi-objective planning model using a weighing approach and a constraint approach.

   (b) Define the efficient frontier. This requires a plot of the feasible combinations of X_1 and X_2.

   (c) Assume that various values are assigned to a weight W_1 for Z_1 whereas weight W_2 for Z_2 is constant and equal to 1, verify the following solutions to the weighing model.

W_1	X_1	X_2	Z_1	Z_2
>6	4	0	16	-8
6	4	0 to 12	16 to 14	-8 to 4
<6 to >1.6	4	2	14	4
1.6	4 to 0	2 to 6	14 to –6	4 to 36
<1.6	0	6	-6	36

2. Let objective $Z_1(\mathbf{X}) = 5X_1 - 2X_2$ and objective $Z_2(\mathbf{X}) = -\,X_1 + 4\,X_2$. Both are to be minimized. Assume that the constraints on variables X_1 and X_2 are:

   i. $-X_1 + X_2 \le 3$

   ii. $X_1 \le 6$

   iii. $X_1 + X_2 \le 8$

   iv. $X_2 \le 4$

   v. $X_1, X_2 \ge 0$

   (a) Graph the Pareto-optimal or non-inferior solutions in decision space.

   (b) Graph the efficient combination of Z_1 and Z_2 in objective space.

(c) Reformulate the problem to illustrate the weighting method for defining all efficient solutions of part (a) and illustrate this method in decision and objective space.

(d) Reformulate the problem to illustrate the constraint method of defining all efficient solutions of part (a) and illustrate this method in decision and objective space.