Module 6:

1. The following matrix displays the joint probabilities of different weather conditions and of different recreation benefit levels obtained from use of a reservoir in a state park:

	POSSIBLE RECREATION BENEFITS		
Weather	RB_1	RB_2	RB_2
Wet	0.10	0.20	0.10
Dry	0.10	0.30	0.20

   (a) Compute the probabilities of recreation levels RB_1, RB_2, RB_3, and of dry and wet weather.

   (b) Compute the conditional probabilities $P(\text{wet} \mid RB_1)$, $P(RB_3 \mid \text{dry})$, and $\mathrm{P}(RB_2 \mid \text{wet})$.

2. In flood protection planning, the 100-year flood, which is an estimate of the quantile $x_{0.99}$, is often used as the design flow. Assuming that the floods in different years are independently distributed:

   (a) Show that the probability of at least one 100-year flood in a 5-year period is 0.049.

   (b) What is the probability of at least one 100-year flood in a 100-year period?

   (c) If floods at 1000 different sites occur independently, what is the probability of at least one 100-year flood at some site in any single year?

3. Assume that annual streamflow at a gauging site have been grouped into three categories or states. State 1 is 5 to 15 m^3/s, state 2 is 15 to 25 m^3/s, and state 3 is 25 to 35 m^3/s, and these grouping contain all the flows on records. The following transition probabilities have been computed from record:

P_{ij}	*1*	$j =$ *2*	*3*
1	0.5	0.3	0.2
$i = 2$	0.3	0.3	0.4
3	0.1	0.5	0.4

   (a) If the flow for the current year is between 15 and 25 m^3/s, what is the probability that the annual flow 2 years from now will be in the range 25 to 35 m^3/s?

(b) What is the probability of a dry, an average, and a wet year many years from now?

4. A Markov chain model for the streamflows in two different seasons has the following transition probabilities

STREAMFLOW IN SEASON 1	STREAMFLOW next Season 2		
	0-3 m^3/s	3-6 m^3/s	$\geq$ 6 m^3/s
0-10 m^3/s	0.25	0.50	0.25
$\geq$ 10 m^3/s	0.05	0.55	0.40

STREAMFLOW IN SEASON 2	STREAMFLOW next Season 1	
	0-10 m^3/s	$\geq$ 10 m^3/s
0-3 m^3/s	0.70	0.30
3-6 m^3/s	0.50	0.50
$\geq$ 6 m^3/s	0.40	0.60

Calculate the steady-state probabilities of the flows in each interval in each season.

5. Assume that the streamflow Q at a particular site has cumulative distribution function $F_Q(q) = q/(1 + q)$ for $q \geq 0$. The withdrawal x at that location must satisfy a chance constraint of the form $\Pr[x \geq Q] \leq 1 - \alpha$. Write the deterministic equivalent for each of the following chance constraints:

$\Pr[x \leq Q] \geq 0.90$ $\qquad$ $\Pr[x \geq Q] \leq 0.80$

$\Pr[x \leq Q] \leq 0.95$ $\qquad$ $\Pr[x \leq Q] \leq 0.10$

$\Pr[x \geq Q] \geq 0.75$

6. Assume that there exist two possible discrete flows Q_{it} into a small reservoir in each of two periods t each year having probabilities P_{it}. Find the steady-state operating policy (release as a function of initial reservoir volumes and current period's inflow) for the reservoir that minimizes the expected sum of squared deviations from storage and release targets. Limit the storage volumes to integer values that vary from 3 to 5. Assume a storage volume target of 4 and a release target of 2 in each period t. (Assume only integer values of all states and

decision variables and that each period's inflow is known at the beginning of the period.) Find the annual expected sum of squared deviations from the storage and release targets.

Period, t	FLOWS, Q_{it}		PROBABILITIES, P_{it}	
	$i = 1$	$i = 2$	$i = 1$	$i = 2$
1	1	2	0.17	0.83
2	3	4	0.29	0.71